

# The apsidal motion in binary stars.

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## Abstract

It is usually accepted to consider an apsidal motion in binary stars as a direct confirmation that a substance inside stars is not uniformly distributed. It is shown in this paper that the apsidal motion in binary systems observation data is in a good agreement with an existence of uniform plasma cores inside stars if they consist of hydrogen-deuterium-helium mixture.

PACS: 64.30.+i; 95.30.-k; 97.10.-q

## 1

An apsidal motion (an advance of periastron) of close binary stars is a result of their non-keplerian moving. It originates from a non-spherical form of stars. The last has been produced by a rotation of stars or their mutual tidal effects. The traditional approach to the estimation of a value of this effect needs to suppose some redistribution of mass inside stars to reach an accordance between the measured data and calculations. Usually it is necessary to assume that the density of a substance at the central region of a star is hundred times greater than a mean density of a star [1], [2].

On the face of it, the model of constant parameter [3],[4], where density inside a star core is constant, is in direct contradiction with the data of observation of an apsidal motion in close binary systems [5]. However, the traditional solution of the task about an apsidal motion is obtained for electrically non-charged matter. It is obvious that for electrically charged matter this solution must be reconsidered.

Let us estimate a contribution of inherent rotation of stars in an apsidal motion. According to [1]-[2], the ratio of  $\omega$  the angular velocity of an apsidal motion and  $\Omega$  the angular velocity of rotation of a star about its axis is

$$\frac{\omega}{\Omega} = \frac{3}{2} \frac{(I_A - I_C)}{Ma^2}, \quad (1)$$

where  $I_A$  and  $I_C$  are momenta of inertia relative to principal axes of the ellipsoid:

$$I_A - I_C = \frac{M}{5}(a^2 - c^2), \quad (2)$$

where  $a$  and  $c$  are polar and equatorial radii of stars.

Thus

$$\frac{\omega}{\Omega} = \frac{3}{10} \frac{(a^2 - c^2)}{a^2}. \quad (3)$$

It is correct when an angular velocity on the ellipse coincides with  $\Omega$  an angular velocity of rotation of a star about its axis .

In the absence of rotation, the equilibrium equation of plasma inside a star is [4]:

$$\gamma g_G + \rho_G E_G = 0 \quad (4)$$

where  $g_G$ ,  $\rho_G$  and  $E_G$  are acceleration of gravitation, gravity-induced density of charge and intensity of gravity-induced electric field. ( $div g_G = 4\pi G \gamma$ ,  $div E_G = 4\pi \rho_G$  and  $\rho_G = \sqrt{G}\gamma$ ).

## 2

One can suppose that at rotation, an additional electric charge with density  $\rho_\Omega$  and electric field  $E_\Omega$  can arise under action of a rotational acceleration  $g_\Omega$ , and the equilibrium equation takes the form:

$$(\gamma_G + \gamma_\Omega)(g_G + g_\Omega) = (\rho_G + \rho_\Omega)(E_G + E_\Omega), \quad (5)$$

where

$$div (E_G + E_\Omega) = 4\pi(\rho_G + \rho_\Omega) \quad (6)$$

or

$$\text{div } E_{\Omega} = 4\pi\rho_{\Omega}. \quad (7)$$

We can seek a solution for electric potential as an expansion on spherical functions. Assuming that an eccentricity is small, we can limit by a second term of the expansion:

$$\varphi = C_{\Omega} r^2(3\cos^2\theta - 1) \quad (8)$$

or in Cartesian coordinates

$$\varphi = C_{\Omega}(3z^2 - x^2 - y^2 - z^2), \quad (9)$$

where  $C_{\Omega}$  is a constant.

Thus

$$E_x = 2 C_{\Omega} x, \quad E_y = 2 C_{\Omega} y, \quad E_z = -4 C_{\Omega} z \quad (10)$$

and

$$\text{div } E_{\Omega} = 0 \quad (11)$$

we obtain important equations:

$$\rho_{\Omega} = 0; \quad (12)$$

$$\gamma g_{\Omega} = \rho E_{\Omega}. \quad (13)$$

Since a centrifugal force must be counterbalanced by the electric force

$$\gamma \Omega^2 x = \rho 2C_{\Omega} x \quad (14)$$

the constant is

$$C_{\Omega} = \frac{\gamma \Omega^2}{2\rho} = \frac{\Omega^2}{2\sqrt{G}} \quad (15)$$

The potential of a positively uniformly charged ball is

$$\varphi(r) = \frac{Q}{R} \left( \frac{3}{2} - \frac{r^2}{2R^2} \right) \quad (16)$$

A negative surface charge induces inside the sphere an electric potential

$$\varphi(R) = -\frac{Q}{R}, \quad (17)$$

where according to [4]  $Q = \sqrt{G}M$ ,  $M$  is the mass of a star.

Thus, the total potential inside the star under consideration is

$$\varphi_\Sigma = \frac{\sqrt{G}M}{2R} \left(1 - \frac{r^2}{R^2}\right) + \frac{\gamma\Omega^2}{2\rho_G} r^2 (3\cos^2\theta - 1) \quad (18)$$

The electric potential must be equal to zero on the surface of a star, since at  $\varphi_\Sigma = 0$  and at  $r = R$  we obtain

$$R^2 - R_0^2 = \frac{\gamma\Omega^2}{\frac{4\pi}{3}\rho^2}, \quad (19)$$

where  $R_0$  is the radius of a star without rotation. This equation describes the form of a rotating star. From this equation

$$a^2 - c^2 = \frac{\Omega^2}{G\gamma} \frac{9}{4\pi}. \quad (20)$$

and Eq.(3) gives

$$\frac{\omega}{\Omega} = \frac{27}{40\pi} \frac{\Omega^2}{G\gamma}. \quad (21)$$

When both stars of a close pair induce an apsidal motion, this equation transforms to

$$\frac{\omega}{\Omega} = \frac{27}{40\pi} \frac{\Omega^2}{G} \left( \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \right), \quad (22)$$

where  $\gamma_1$  and  $\gamma_2$  are densities of stars.

### 3

A density of non-degenerate non-relativistic plasma can be determined in the following way. At high density and temperature an electron gas is almost ideal [6] and the ideal gas law will serve as its equation of state in first

approximation. It needs to take to account two corrections to describe electron gas properties more exactly - a correction for identity of electrons and a correction for a presence of positively charged nuclei. The first of them is positive, it increases the incompressibility of electron gas. The second correction decreases its incompressibility and it is negative. These correction are known [6], and that is why with allowance for both corrections the free energy of electron gas takes the form

$$F = F_{ideal} + N \frac{\pi^{3/2} e^3 a_0^{3/2}}{4(kT)^{1/2}} n - N \frac{2e^3 Z^3}{3} \frac{\pi^{1/2}}{(kT)^{1/2}} n^{1/2}, \quad (23)$$

where  $Z$  is the nucleus charge,  $a_0 = \frac{\hbar^2}{me^2}$  is the Bohr radius.

At a constant full number of particles in the system and at constant temperature, the free energy minimum equation is

$$\left( \frac{\partial F}{\partial n} \right)_{N,T} = 0, \quad (24)$$

what allows one to obtain the steady-state value of density of particles of hot non-relativistic plasma

$$n_0 = \frac{16Z^6}{9\pi^2 a_0^3} \simeq 2 \cdot 10^{24} Z^6 \text{ cm}^{-3}. \quad (25)$$

Thus, the equilibrium density of a star is

$$\gamma = \frac{16Am_p Z^5}{9\pi^2 a_0^3} \quad (26)$$

where  $A$  is the mass number,  $m_p$  is proton mass.

## 4

If we introduce  $P = \frac{2\pi}{\Omega}$  the period of ellipsoidal rotation and  $U = \frac{2\pi}{\omega}$  the period of the apsidal motion, we obtain from Eq.(22) the main equation of our theory:

$$\frac{P^3}{U} \xi = \left( \frac{1}{A_1 Z_1^5} + \frac{1}{A_2 Z_2^5} \right), \quad (27)$$

where the constant

$$\xi = \frac{243 \pi^3 a_0^3}{160 G m_p} \text{ sec}^{-2}. \quad (28)$$

For hydrogen the value  $1/AZ^5$  is equal to 1, for deuterium 0.5, for helium 1/128. Thus, for two stars both consisting of hydrogen the right part of Eq.(27) is equal to 2. For pairs consisting of hydrogen-deuterium it is 1.5. For D-D pairs and H-He pairs it equals to 1. For D-He pairs 0.5, and for He-He pairs 1/64.

Thus, for pairs of a star consisting of hydrogen, deuterium and helium

$$\frac{1}{64} < \frac{P^3}{U} \xi < 2. \quad (29)$$

The periods  $U$  and  $P$  are measured for few tens of stars [5] and we can compare our calculation with the data of these measurements. The distribution of binary stars by the value of  $P^3\xi/U$  is shown on Fig.1 in logarithmic scale. On upper abscissa the value of  $\log(\frac{1}{A_1 Z_1^5} + \frac{1}{A_2 Z_2^5})$  is shown. The lines mark the values of parameters for different pairs of binary stars.

It can be seen that all measured data (with the exception of few pairs only) are limited by factors 1 and 1/64. This argues for adequate interpretation and satisfactory accuracy of our estimations of the effect of the apsidal motion.

The interesting result can be obtained if we additionally take in to account the measured data on mass of rotating pairs. Accordingly [4], the star mass is determined by the  $A/Z$  ratio:

$$\frac{A}{Z} = \sqrt{\frac{12M_\odot}{M_\star}} \quad (30)$$

and from Eq.26

$$A Z^5 = \frac{\gamma_\star 9\pi^2 a_0^3}{16m_p} \quad (31)$$

At solution of this equations with using of data observation of  $M_\star$  and  $\gamma_\star$  from the apsidal motion we can obtain a distribution of stars on  $Z$  and  $A$  separately (Fig.2). As the periods of the apsidal motion of stars consisting of nuclei heavier than helium are more than  $10^5$  years and rather are immeasurable, this distribution is cut off on  $1 < Z < 2$  and  $1 < A < 3$ .

Figure 1: The distribution of binary stars on value of  $P^3\xi/U$ .

Figure 2: The distribution of binary stars on values A and Z.

This distribution rather does not give a new astrophysical information, but it illustrates an efficiency and a cardinal utility of developed consideration.

## References

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